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Microwave Measurement of the Temperature Coefficient of Permittivity for Sapphire and Alumina

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Abstract—**Measurements of the temperature coefficients of permittivity and of thermal expansion, for the important MIC substrate materials alumina and sapphire, are reported. The results are presented and in the case of sapphire include figures for the two main crystal orientations. An interesting correlation exists between our results for alumina substrates and those for sapphire substrates in which the optical axis is perpendicular to the plane of the slice. The temperature stability of resonators on sapphire and alumina is discussed and experimental data are presented.**

INTRODUCTION

Knowledge of the temperature dependence of the properties of microwave integrated circuit (MIC) substrate materials can be as important as precise knowledge of the properties themselves. Recently, we presented [1] a technique for precise measurement of substrate permittivity. In this short paper we describe the measurement of the temperature coefficient of permittivity for sapphire and alumina substrates by a refinement of the earlier method.

The substrates used for this work were 25 mm square by 0.5 mm thick and they were metallized all over to form resonant cavities. For each resonant mode the resonance frequency and the unloaded cavity Q factor were measured as functions of cavity temperature. These data, in conjunction with separate measurement of the substrate expansion coefficients, were used to determine the temperature coefficients of permittivity.

Fig. 1 illustrates one of our cavities which is formed by first metallizing the substrate on all sides and then etching a single slot on one edge for coupling. A standard launcher is brought into contact with the cavity so that the flat tab lies on the broad-face metallization and above the center of the coupling slot. The cavity is held firmly between the launcher tab and a brass block which is screwed to the launcher flange. The $\boldsymbol{E}$ and $\boldsymbol{H}$ fields in the coaxial feed line approximately match the corresponding cavity fields at the slot, and coupling will be by $\boldsymbol{E}$ and/or $\boldsymbol{H}$ field depending on the mode excited. The coupling coefficient is mode dependent.

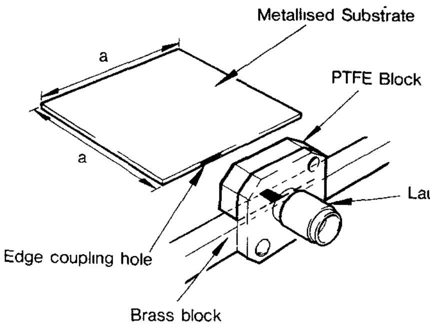

Fig. 1. Metallized-substrate cavity and launcher.

THEORY

If the substrate medium is isotropic then TE modes may be excited in the cavity of Fig. 1 with the $\boldsymbol{E}$ field directed across the thin dimension. The substrate dielectric constant ϵ, substrate dimension a, and mode frequencies $f_{n,m}$ in the range from 2 to 12 GHz are related, to a high degree of accuracy, by (1) in which n and m are integers and c is the velocity of light in vacua

$$\epsilon(f,T) = \frac{c^2}{a^2 f^2_{n,m}} \left\{ \frac{m^2 + n^2}{4} \right\}. \quad (1)$$

If the substrate medium is sapphire the excited modes will still be TE provided one of the major crystal axes is directed across the thin dimension of the slice. The permittivity tensor for these orientations reduces to a diagonal matrix and (1) remains valid; but the permittivity value given by the equation is that parallel to the $\boldsymbol{E}$ field i.e., the permittivity across the thin dimension.

In deriving (1) cavity losses and other second-order effects have been neglected. The frequency-pulling effect of the temperature-dependent cavity losses may be taken into account by considering the resonance frequency f as a function of both the unloaded Q factor, Q_0, and the cavity temperature T, i.e.,

$$f_{n,m} = f(Q_0,T). \quad (2)$$

Differentiating (1) with respect to T and using (2) we may derive the partial differential equation (3) i.e.,

$$\frac{1}{\epsilon}\frac{\delta\epsilon}{\delta T} = -\alpha_1 - \alpha_2 - \frac{1}{f}\frac{df}{dT}\left\{2 + \frac{f}{\epsilon}\frac{\delta\epsilon}{\delta f} - \frac{2(\delta f/\delta Q_0)(dQ_0/dT)}{df/dT}\right\}. \quad (3)$$

The constants α_1 and α_2 are the thermal expansion coefficients $(1/a)(da/dT)$ for the sides "a" in Fig. 1, which may be different for anisotropic materials, and in separating these in (3) we have assumed that they are not greatly different. The $\delta\epsilon/\delta f$ term may be neglected provided that it is small compared with the last (Q_0) term. A straightforward comparison of the two terms using representative experimental data yields this condition as

$$\frac{1}{\epsilon}\frac{\delta\epsilon}{\delta f} \ll 0.01/\text{GHz}.$$

Although we have been unable to find a published figure over our range of temperatures and frequencies, [2] implies there is no variation at all in the frequency coefficient below 400°C. In the absence of any evidence to the contrary, we therefore feel justified in neglecting it. The factor $\delta f/\delta Q_0$ is calculable [3] and is given by

$$\frac{\delta f}{\delta Q_0} = \frac{f}{2Q_0^2}. \quad (4)$$

Manuscript received April 26, 1974; revised August 12, 1974.
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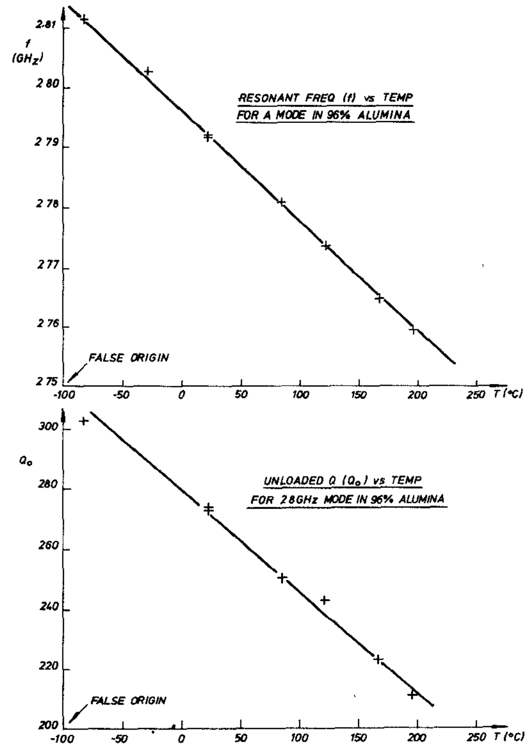


Fig. 2. f/T and Q_0/T for mode at 2.8 GHz in 96 percent alumina.

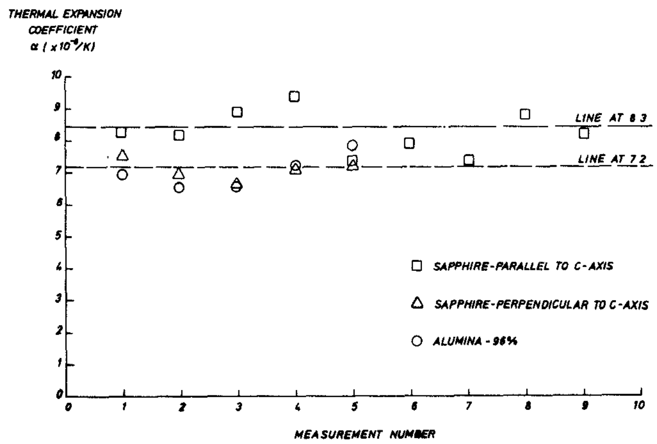


Fig. 3. Thermal expansion coefficient measurements for sapphire and 96 percent alumina.

Equation (3) may therefore be simplified to

$$\frac{1}{\epsilon}\frac{\delta\epsilon}{\delta T} = -\alpha_1 - \alpha_2 - \frac{2}{f}\frac{df}{dT} + \frac{1}{Q_0^2}\frac{dQ_0}{dT}. \quad (5)$$

The left side of (5) is the required temperature coefficient of permittivity and its evaluation demands measurement of $\alpha_1(T)$, $\alpha_2(T)$, $f(T)$, and $Q_0(T)$.

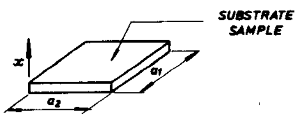
It is worth remarking that if the same (n,m) mode is excited in two or more metallized substrates of different size it may be practicable to extract $(1/\epsilon)(\delta\epsilon/\delta f)$. The present measurements are, however, restricted to substrates of one size.

MEASUREMENTS

In the present work on sapphire we have used slices with the optical axis parallel to the plane of the slice and with it perpendicular to the plane of the slice. We have results therefore for both crystal orientations. The slices were supplied to a ±2° orientation accuracy. The resonance frequencies and unloaded Q factors were obtained using a network analyzer system arranged to measure reflection coefficient. An oven was used to achieve temperatures up to 200°C and a flask of cold methanol to achieve temperatures down to −80°C. Typical laboratory measurements are shown in Fig. 2. Straight lines were the best fit on our experimental data for $f(T)$ and $Q_0(T)$ and the slopes of these lines were computed by a least squares method. The terms $(1/f)(df/dT)$ and $(1/Q_0^2)(dQ_0/dT)$ were evaluated at 70°C. The thermal expansion coefficients were measured using a mechanical system with data analysis to separate instrument and substrate expansions. Fig. 3 indicates the spread in our thermal expansion measurements.

In (5) the frequency term is dominant, the thermal expansion and Q-factor terms being essentially corrections each amounting to less than 10 percent of the final figure for $(1/\epsilon)(\delta\epsilon/\delta T)$. Consequently great accuracy is not essential in the thermal expansion and Q-factor measurements. A figure for $(1/\epsilon)(\delta\epsilon/\delta T)$ can be obtained from measurements on only one mode but, in the present work, $f(T)$ and $Q_0(T)$ have been measured for many modes between 2 and 12 GHz in each substrate in order to check the self-consistency of the results.

TABLE I
RESULTS



PROPERTY	SAPPHIRE C- AXIS PARALLEL TO SIDE a_1	SAPPHIRE C- AXIS PERPENDICULAR TO SIDES a_1 AND a_2	ALUMINA (96%) SAMPLE 1	ALUMINA (96%) SAMPLE 2
ϵ_x	9.34	11.49	8.76	8.76
$\frac{1}{\epsilon_x}\frac{\partial \epsilon_x}{\partial T}$	7.5×10^{-5}/K [14; 16]	12.6×10^{-5}/K [14; 2.4]	11.0×10^{-5}/K [10; 1.8]	11.0×10^{-5}/K [13; 6]
$\alpha_1 = \frac{1}{a_1}\frac{da_1}{dT}$	8.3×10^{-6}/K	7.2×10^{-6}/K	7.2×10^{-6}/K	

TABLE II
TEMPERATURE COEFFICIENT OF FREQUENCY FOR RESONATORS ON SAPPHIRE (C AXIS PERPENDICULAR TO PLANE OF SLICE)

Resonator Type	$\frac{1}{f}\frac{df}{dT}$ (10^{-5} K^{-1})
linear 50Ω microstripline	6.2
thin-ring microstripline	5.9 - 6.3
thick-ring	7.0 - 7.2
flat-disc	6.8 - 7.2

Note: Each resonator was measured over numerous modes in the frequency range 2–12 GHz. The spread in values is indicated. df/dT is negative.

RESULTS

The results are presented on Table I which also displays the permittivities reported in [1]. The expansion coefficients are accurate to about ±5 percent. The figures shown, in square brackets, below the temperature coefficients of permittivity are the number of modes measured and the percentage rms deviation of the individual results as derived from the various modes. Thus, for sapphire the quoted temperature coefficient of permittivity in the direction perpendicular to the optical axis is the mean of the results obtained from 14 different resonance modes and the rms deviation of the individual results was 16 percent. The results are based on measurements on only a few substrates. Whereas this is acceptable for the crystalline material (sapphire), for alumina there will usually be a spread in the properties between different samples and manufacturers.

An interesting correlation has emerged between the results obtained for our alumina substrates and those for sapphire substrates with the optical axis directed across the thin dimension of the slice. For these substrates the expansion coefficients were the same (within the accuracy of the measurement) and the temperature coefficients of permittivity were well correlated, the value for alumina being less than the value for sapphire. (The presence of air and other impurities in the alumina, responsible [4] for the difference between the permittivities of alumina and pure sapphire, might be expected to result in a similar difference between the temperature coefficients of permittivity for alumina and sapphire). The results suggest there may be a significant platelet orientation and corresponding anisotropy in our alumina and, apparently, a platelet orientation may indeed be expected depending on the method of manufacture of the alumina. A recent paper [5] has drawn attention to anisotropic effects in alumina substrates.

TEMPERATURE STABILITY OF RESONATORS ON SAPPHIRE AND ALUMINA

An expression for the temperature coefficient of resonance frequency $(1/f)(df/dT)$ for the metallized-substrate resonators used in the present work may be obtained by transposing in (5) i.e.,

$$\frac{1}{f}\frac{df}{dT} = -\frac{1}{2}\left(\frac{1}{\epsilon}\frac{\delta\epsilon}{\delta T} + \alpha_1 + \alpha_2\right) + \frac{1}{2Q_0^2}\frac{dQ_0}{dT}. \qquad (6)$$

The contribution due only to the temperature coefficient of substrate permittivity is

$$\frac{1}{f}\frac{\delta f}{\delta T} = -\frac{1}{2}\frac{1}{\epsilon}\frac{\delta\epsilon}{\delta T}$$

and, for a sapphire substrate in which the C axis is perpendicular to the plane of the slice (i.e., $\boldsymbol{E}$ field parallel to C axis), this has the value -6.3×10^{-5}/K. This is the predominant effect in various resonators on sapphire as may be seen from Table II in which are presented the measured frequency coefficients for resonators produced in the "microstrip configuration", i.e., copper ground plane on one side of the substrate and resonator pattern (line, ring, or disk) on the other. There was a nonsystematic variation of $(1/f)(df/dT)$ with the frequency of the mode excited and Table II indicates the spread in the values.

For the flat-disk resonator most of the $\boldsymbol{E}$ field is in the substrate and parallel to the C axis and, for this orientation, $\alpha_1 = \alpha_2 = 7.2\times 10^{-6}$/K. Equation (6) is a good approximation for this case and, neglecting the Q_0 term, yields $(1/f)(df/dT) = -(6.3\times 10^{-5} + 7.2\times 10^{-6}) = -7\times 10^{-5}$/K.

For the open-structure microstripline resonator the "effective permittivity" for the line is a function of the air and substrate permittivities and of the geometry of the line. Equation (6) clearly requires modification for this case. Qualitatively, however, since $(1/\epsilon)(\delta\epsilon/\delta T)$ for air is relatively small and negative an improvement in resonator temperature stability is expected. (From information in [6] it may be deduced that for air at 20°C and at constant pressure of 1 atm $(1/\epsilon)(\delta\epsilon/\delta T) = -1.8 \times 10^{-6}$/K.) The improvement, from 7 to 6 parts in 10^6/K as indicated in Table II, is fairly small since for microstrip most of the E field is still in the substrate.

The data in Table I may be used to predict the frequency coefficients for resonators on either orientation of sapphire, or our 96 percent alumina. For resonators on our alumina the frequency coefficients are about 10 percent lower than those indicated in Table II for sapphire, but as noted previously some variation is expected in the figures for alumina depending on the impurity content and the method of manufacture.

ACKNOWLEDGMENT

The authors wish to thank E. England for producing the substrate cavities, M. Matthews for assistance with the thermal expansion measurements, and C. Garrett for her work on the data analysis. Useful discussions with Dr. G. J. Gittens on the chemistry of alumina are also gratefully acknowledged.

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Electromagnetic Power Absorption in Anisotropic Tissue Media

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***Abstract*—Strong dielectric-constant anisotropy exists in muscle tissue at the lower microwave frequencies. Based on a model derived from tissue measurements, an analysis is carried out for single and multiple tissue layers. Calculated effects of tissue anisotropy on microwave fields and power absorption in the tissues are presented.**

Manuscript received July 19, 1974; revised November 11, 1974. This work was supported by the USAF School of Aerospace Medicine, Brooks Air Force Base, Texas.
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At frequencies below 100 Hz it is known that a strong conductance anisotropy exists in muscle tissue [1]. A fifteen-to-one difference in skeletal muscle resistivity has been measured at ECG frequencies. Schwan [2] attributes tissue permittivity and conductivity relaxation in the 1-MHz region to cell-wall polarization effects. This implies that relaxation effects exist when field components are normal to cell walls, but are not present with field components parallel to cell walls. In the work reported here, an idealized anisotropic tissue medium is assumed, consisting of infinitely long perfectly parallel muscle fibers generating relaxation effects only when there are E-field components perpendicular to the fibers. Based on this model, a theory is developed for field effects in anisotropic tissue, and equations for power absorption are derived.

Tissue data in the frequency range 0.001–100 MHz are based on data by Rush *et al.* [1], Schwan [2], and Johnson and Guy [3]. The data estimated from these references are summarized in Tables I and II for anisotropic skeletal muscle and fat.

Since the muscle medium consists of anisotropic conductivity and permittivity, a complex tensor permittivity is used

$$\bar{\epsilon} = \begin{vmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{vmatrix}.$$

The vector wave equation for the electric field in an anisotropic medium is derived from Maxwell's equations

$$\nabla^2 E - \nabla(\nabla \cdot E) + \omega^2\mu_0 \bar{\epsilon} \cdot E = 0.$$

When there are no transverse variations in the fields, as for plane-wave propagation in the z direction, the wave equation reduces to

$$(\partial^2 E/\partial z^2) + \omega^2\mu_0 \bar{\epsilon} \cdot E = 0.$$

The solution to this second-order differential equation is

$$E = E_x \hat{x} \exp(-jk_x z) + E_y \hat{y} \exp(-jk_y z)$$

TABLE I
ESTIMATED CONDUCTIVITY AND DIELECTRIC CONSTANT VARIATIONS IN SKELETAL MUSCLE IN THE y DIRECTION PERPENDICULAR TO THE MUSCLE FIBERS

f(MHz)	σ_y(mhos/m)	ϵ_y/ϵ_0
.001	.05	125,000
.01	.08	75,500
.1	.30	19,000
1.	.56	1,970
10.	.56	252
100.	.67	84.

Note: In the x direction, parallel to the fibers, we assume $\sigma_x = 0.67$ mho/m and $\epsilon_x/\epsilon_0 = 84$.

TABLE II
ESTIMATED CONDUCTIVITY AND DIELECTRIC CONSTANT VARIATIONS IN FAT

f(MHz)	σ_f(mhos/m)	ϵ_f/ϵ_0
.001	.028	50,000
.01	.031	20,000
.1	.033	4,000
1.	.036	314
10.	.038	30
100.	.040	7.5